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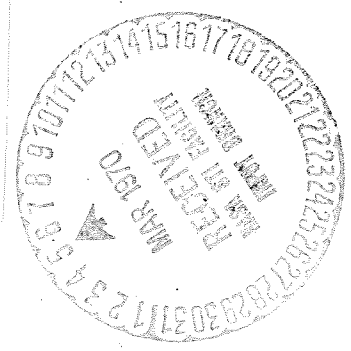

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OPTIMIZATION OF NONLINEAR CHARACTERISTICS

(K. Inan and C. A. Desoer)

On the problem of optimization of Nonlinear Characteristics, the derivation of the necessary conditions has been completed. The results were presented at the Allerton Conference (1969) and the detailed derivations were the subject of the Electronics Research Laboratory Technical Memorandum No. ERL-M267. A paper on the subject has been accepted for publication by the Journal of Math Analysis and Application. We are now developing an algorithm for the iterative calculation of the optimum nonlinearity.

[1] K. Inan and C. A. Desoer, "Optimization of nonlinear characteristics," Proc. 7th Allerton Conference, October 1969, pp. 105-108.

MULTIPLE-INPUT MULTIPLE-OUTPUT DISCRETE SYSTEMS

(C. A. Desoer)

The work on the Input-Output properties of Discrete System has been submitted for publication in the Journal of the Franklin Institute and has been presented at the Allerton Conference. In order to tie this work with some recent work on stabilization we are presently investigating some ways to remove certain assumptions that were necessary in our 1969 derivation.

[1] C. A. Desoer and M. Y. Wu, "Input-output properties of multiple-input multiple-output discrete systems. Part I and II, Proc. 7th Allerton Conference, October 1969.

DIFFERENTIAL GAMES

(P. Varaiya)

P. Varaiya has just established existence of equilibrium solutions for N-person differential games when the dynamics are linear and the cost to each player is convex. It is shown that under certain mild assumptions on the costs there always exists an open-loop equilibrium solution at least on a sufficiently small interval. It is also shown that these results cannot be extended. The results are given in Ref. 1. The next step in the completion of this theory is to study the game when closed-loop strategies are allowed. We shall also give a procedure to calculate the equilibrium strategies.

[1] N-person non-zero sum differential games with linear dynamics (to appear in J. SIAM Control).

ACCELERATION TECHNIQUES FOR OPTIMIZATION ALGORITHMS: PART I

(K. Jeyarasasingam and E. Polak)

This part of the project, which is primarily concerned with nonlinear programming and discrete optimal control algorithms is being continued. At present computer programs are being written which will enable us to evaluate some of the acceleration techniques under consideration.

ACCELERATION TECHNIQUES FOR OPTIMIZATION ALGORITHMS: PART II

(R. Klessig and E. Polak)

This part of the project is primarily concerned with continuous optimal control algorithms. In particular, we are investigating methods

for inserting adaptive subprocedures into algorithms, such as that of steepest descent, which automatically adjust the required integration step size and order of the integration formula, at each integration. This results in algorithms that are convergent and that integrate very crudely, and hence very rapidly, when one is far from an optimal solution, and increase their accuracy of integration as an optimal solution is approached. A certain amount of numerical experimentation has been performed and leads us to expect that these adaptive algorithms will result in considerable savings of computer time, as compared to traditional algorithms. In addition, we are developing new families of algorithms which are not simply efficient elaborations of existing ones.

DECOMPOSITION ALGORITHMS FOR OPTIMAL CONTROL

(G. Meyer and E. Polak)

This study has been completed and the resulting paper, entitled "A Decomposition Algorithm for Solving a Class of Optimal Control Problems", has been accepted for publication in the Journal of Mathematical Analysis and Applications.

ABSTRACT MODELS FOR THE SYNTHESIS OF OPTIMIZATION ALGORITHMS

(G. Meyer and E. Polak)

The convergence of optimization algorithms has been studied extensively in recent years (see [1], [2], [3], [4], [5], [6]). The approach generally adopted in these studies consisted of defining a class of algorithms and then giving convergence theorems which applied to every

algorithm in this class. This approach has resulted in the development of general procedures which have considerably simplified the task of establishing whether an algorithm is convergent. However, the emphasis so far has been on analysis. Very few of the existing results provide guidelines for the synthesis of algorithms. Our approach to the problem of synthesis consists of two parts. First we develop abstract models for algorithms. These models guide the inventive process towards "conceptual" algorithms which will later be easily made implementable. Once the abstract models are established, we obtain a set of methods for converting "conceptual" algorithms, falling into the class defined by the abstract models, into "implementable" iterative procedures.

The results obtained so far, have been described in the Electronics Research Laboratory Technical Memorandum No. ERL-M268, entitled "Abstract Models for the Synthesis of Optimization Algorithms," and this memorandum has been submitted for publication in SIAM Journal on Control.

The two most important aspects of the theory discussed in Memorandum No. ERL-268 are the following:

(i) By using abstract models one can separate out the essential properties of an algorithm from the non essential ones.

(ii) Given a "conceptual" algorithm in which one has to perform in sequence several operations each of which requires an infinite number of iterations, one can obtain an "implementable" algorithm by shuttling between these infinite operations, combining them into a single infinite operations.

[1] B. T. Polyak, "Gradient Methods for the Minimization of Functionals,"

USSR Computational Mathematics and Mathematical Physics, Vol. 3, No. 4 (1963), pp. 864-878. (Translation of Zh. Vychisl. Mat. i Mat. Fiz., Vol. 3, No. 4 (1963), pp. 643-653.)

[2] W. I. Zangwill, "Convergence Conditions for Nonlinear Programming Algorithms," Working Paper No. 197, Center for Research in Management Science, University of California, Berkeley, November 1966.

[3] D. M. Topkis and A. Veinott, "On the Convergence of Some Feasible Directions Algorithms for Nonlinear Programming," J. SIAM Control, Vol. 5, No. 2 (1967), pp. 268-279.

[4] E. Polak, "On Primal and Dual Methods for Solving Discrete Optimal Control Problems," Proceedings, Second International Conference on Computing Methods in Optimization Problems, San Remo, Italy, Sept. 9-13, 1968, Academic Press, 1969.

[5] E. Polak, "On the Convergence of Optimization Algorithms," Revue Francaise d' Informatique et de Recherche Operationelle, Serie Rouge, No. 16, 1969, pp. 17-34.

[6] W. I. Zangwill, Nonlinear Programming: A Unified Approach, Prentice Hall Inc., Englewood Cliffs, N. J., 1969.

DECISION-MAKING IN A FUZZY ENVIRONMENT

(L. A. Zadeh)

By decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control, can be represented as fuzzy sets. For example, a constraint on a non-negative variable x which is expressed as "x should not be substantially in excess of 10," is a fuzzy constraint in which the source of fuzziness is the word "substantially." In this case, the constraint may be represented as a fuzzy set C whose membership function $\mu_C(x)$ is given subjectively by, say, $\mu_C(x) = 1$ for $0 \leq x \leq 10$ and $\mu_C(x) = 1 - (1 + (x-10)^{-2})^{-1}$.

If a goal is represented as a fuzzy set G in a space of alternatives

$X = \{x\}$, and a constraint is represented as a fuzzy set C in the same space, then a fuzzy decision D is defined as the intersection of G and C. Thus, the membership function of D is given by

$$\mu_D(x) = \text{Min}(\mu_C(x), \mu_G(x))$$

When faced with the choice of an alternative in X which is a best compromise between the goal and the constraint, it is reasonable to choose that x which has the highest grade of membership in D, that is, an x which maximizes $\mu_D(x)$. Such an x is termed a maximizing decision.

A central problem in decision-making in a fuzzy environment is that of finding a maximizing decision in the case of multistage decision processes. Several problems of this type are considered in a recent NASA report [1] which is scheduled for publication in Management Science. In particular, we have considered systems described by state equations of the form

$$x_{t+1} = f(x_t, u_t) \quad , \quad t = 0, 1, 2, \ldots$$

in which x_t denotes the state at time t and u_t is the input at time t. Both x_t and u_t are assumed to range over finite sets for all t.

The function f may be non-random or random. The fuzzy constraints on the input may be time or state-dependent.

The process is assumed to terminate when x_t enters a prescribed set of states in the state space, with the fuzzy goal assumed to be a fuzzy subset of this set.

It is shown that under these assumptions the determination of a maximizing decision reduces to the solution of a functional equation. A

simple technique for the solution of this equation has been developed.

Work is now in progress on more general types of problems of this kind in which the system under control may be fuzzy. We are also considering pursuit-evasion problems involving fuzzy goals and constraints. The main aim of our research is to develop quantitative as well as qualitative techniques for dealing with decision processes which are too complex or ill-defined to admit of analysis by classical means.

[1] R. E. Bellman and L. A. Zadeh, "Decision Making in a Fuzzy Environment," Report No. ERL-69-8. To appear in Management Science.

SORTING ALGORITHMS FOR PRE-ORDERED STRUCTURES

(D. Chan and L. A. Zadeh)

A pre-ordering on a finite set X is a relation R(x, y) on X which is transitive but not necessarily antisymmetric. Such relations play an important role in optimization under vector-valued criteria. Thus, a point X_0 in X is undominated (noninferior) if there is no y in X such that (x_0, y) is in R. Given R as a set of pairs (X, y), the problem of sorting is that of arranging the elements of X in a tree-like structure, with the tips of the tree corresponding to undominated points. Generalization of the linear sorting algorithms to structures of this type are being investigated and the efficiency of their implementation in FORTRAN IV is studied by computer experimentation.

III. LIST OF REPORTS AND PAPERS

1. E. Polak, "On the evaluation of optimal and nonoptimal control strategies," IEEE Trans. on Automatic Control, Vol. AC-9, No. 2, 1964.

2. D. J. Sakrison, "Efficient estimation of radar astronomy target parameters," presented at the International Conference on Microwaves, Circuit Theory and Information Theory, Tokyo, Japan, September 7-11, 1964.

3. C. A. Desoer, "Some results in stability theory," presented at the Second Annual Allerton Conference on Circuit and Systems Theory, September 28, 1964. (also ERL Int. Tech. Memorandum M98, September 21, 1964).

4. B. W. Jordan and E. Polak, "Theory of a class of discrete optimal control systems," J. Electronics and Control, Vol. 17, No. 6, p. 697, December 1964

5. E. Polak, Review of A. A. Fel'dbaum, "Fundamentals of the theory of optimal control," Fizmatgiz, Moscow 1963, Mathematical Reviews, Vol. 29, No. 2, pp. 404-405, February, 1965.

6. L. A. Zadeh, "Fuzzy sets and systems," presented at the Symposium of System Theory, Polytechnic Institute of Brooklyn, April 20, 1965, published in the Proceedings of the Symposium on System Theory, pp. 29-37, April 1965.

7. C. A. Desoer, "A generalization of Popov's criterion, "IEEE Trans. on Automatic Control, Vol. AC-10, pp. 182-185, April 1965.

8. C. A. Desoer, "A stability criterion obtained by a method of comparison," IEEE Trans. on Automatic Control, Vol. AC-10, pp. 185, April 1965.

9. P. P. Varaiya, "Decomposition of large-scale systems," presented at the First International Conference on Programming and Control, Air Force Academy, Colorado, April 1965, J. SIAM on Control, Vol. 4, No. 1, pp. 173-178, 1966.

10. D. J. Sakrison, "Processing of data generated by sensors moving in a varying field," presented at the Fifth Annual Cospar Conference, Buenos Aires, May 1965, and appearing in the sixth volume of Space Research, Interscience, copyright date 1966, (also ERL Int. Tech. Memorandum M113, March 19, 1965).

11. D. J. Sakrison, "Efficient recursive estimation; application to estimating the parameters of a covariance function," International Journal of Engineering Science, Vol. 3, pp. 461-483, 1965, (also ERL Int. Tech. Memorandum M76, July 2, 1964).

12. B. W. Jordan and E. Polak, "Optimal control of aperiodic discrete-time systems," Journal SIAM Control, Ser A, Vol. 2, No. 3, pp. 332-343, April 1965.

13. M. D. Canon and E. Polak, "Analog circuits for energy and fuel optimal control and linear discrete systems," ERL Int. Tech. Memorandum M95, August 24, 1964.

14. C. D. Cullum and E. Polak, "On the classification of optimal control problems," J. SIAM on Control, Vol. 4, No. 3, pp. 403-420, (also ERL Int. Tech. Memorandum M116, January 15, 1965).

15. B. W. Jordan, H. Halkin, E. Polak and J. B. Rosen, "Theory of optimum discrete time systems," Proc. Third IFAC Congress, London, 1966, paper No. 2 PB., (also ERL Int. Tech. Memorandum M119, February 15, 1965).

16. M. D. Canon and J. H. Eaton, "A new algorithm for a class of quadratic programming problems with applications to control problems," presented at the First International Conference on Programming and Control, Air Force Academy, Colorado, April 1965, Journal SIAM Control, Vol. 4, No. 1, pp. 34-45, 1966.

17. J-P Jacob and E. Polak, "On the inverse of the operator $(\cdot) = A(\cdot) + (\cdot)B$," Amer. Math. Monthly, Vol. 73, No. 4, pp. 388-390, April 1966.

18. D. J. Sakrison, "Efficient recursive estimation of the parameters of a radar or radio astronomy target," IEEE Trans. On Information Theory, Vol. 12, No. 1, pp. 35-41, January 1966, (also ERL Int. Tech. Memorandum M110, December 18, 1964).

19. P. P. Varaiya, "An extremal problem in Banach space with application to optimal control," ERL Int. Tech. Memorandum M180, August 12, 1966.

20. E. Wong and E. Eisenberg, "Iterative synthesis of threshold functions," Journal of Mathematical Analysis and Applications, Vol. 11, No. 1-3 pp. 226-235, July 1965.

21. P. P. Varaiya, "Nonlinear programming in Banach space," SIAM J. Appl. Math, Vol. 15, No. 2, March 1967, (also ERL Int. Tech. Memorandum M137, December 10, 1965).

22. C. A. Desoer, P. P. Varaiya, "The minimal realization of a non-anticipative impulse response matrix, "J. App. Math., Vol. 15, No. 3 pp. 754-764, May 1967, (also ERL Int. Tech. Memorandum M138, December 16, 1965).

23. E. Polak, "An algorithm for reducing a linear time invariant differential system to state form, "IEEE Trans. on Automatic Control, Vol. AC-11, No. 3, pp. 577-579, July 1965, (also ERL Int. Tech. Memorandum M140, July 1966).

24. P. P. Varaiya and R. W. Liu, "Normal form and stability of a class of coupled nonlinear networks," IEEE Trans. on Circuit Theory, Vol. CT-13, No. 4, pp. 413-418, December 1966, (also ERL Int. Tech. Memorandum M141, December 15, 1965).

25. C. D. Cullum and E. Polak, "Equivalence relations for the classification and solution of optimal control problems," submitted to SIAM Journal of Control, (also ERL Int. Tech. Memorandum M147, January 24, 1966).

26. P. P. Varaiya and R. W. Liu, "Bounded-input bounded-output stability of nonlinear time-varying differential systems," J. SIAM Control, Vol. 4, No. 4, pp. 698-704, 1966, (also ERL Int. Tech. Memorandum M148, January 25, 1966).

27. M. D. Canon, C. D. Cullum and E. Polak, "Constrained minimization problems in finite dimensional spaces," J. SIAM Control, Vol. 4, No. 3, 1966 (also ERL Int. Tech. Memorandum M149, December 1, 1965).

28. P. P. Varaiya, "An extremal problem in Banach space with applications to discrete and continuous time optimal control," ERL Int. Tech. Memorandum M153, March 8, 1966.

29. E. Polak and A. Larsen, Jr., "Some sufficient conditions for continuous-linear programming problems," Int. J. Engineering Science, Vol. 4, pp. 5839604, 1966, (also ERL Int. Tech. Memorandum M123, October 1, 1965).

30. "Notes on System Theory, Vol. V," ERL Tech. Report No. 64-8, April 1965.

31 "Notes on System Theory, Vol. VI," ERL Tech. Report No. 64-29, August 1964.

32. "Notes on System Theory, Vol. VII," ERL Tech. Report No. 65-14, May 1965.

33. D. Chazan, "Cost-function characterization of systems," ERL Tech. Report No. 66-5, April 21, 1966.

34. C. T. Lee and C. A. Desoer, "Stability of single-loop nonlinear feedback system," Proceedings - 3rd Annual Allerton Conference on Circuit and System Theory, October 20-22, 1965, (also ERL Tech. Report No. 66-13, May 12, 1966).

35. E. Polak and K. Y. Wong, "Identification of linear discrete time systems using the instrumental method," IEEE Trans. Automatic Control, Vol. AC-12, No. 6, Dec. 1967, pp. 707-718, (also ERL Int. Tech. Memorandum M187, Dec. 9, 1966).

36. E. Polak and N. O. DaCunha, "Constrained minimization under vector-valued criteria in finite dimensional spaces, "Journal Mathematical Analysis and Applications, Vol. 19, No. 1, pp. 103-124, July 1967, (also ERL Int. Tech. Memorandum M188, October 31, 1966).

37. C. A. Desoer, "On the problem of finite escape time," ERL Int. Memorandum M190, October 20, 1966.

38. E. Polak, and N. O. DaCunha, "Constrained minimization under vector-valued criteria in linear topological space," Proc. Conference on Math Theory of Control, Los Angeles, pp. 96-109, 1967, (also ERL Int. Memorandum M191, November 4, 1966).

39. J. P. Jacob and E. Polak, "On a class of pursuit-evasion problems," IEEE Trans on Automatic Control, Vol. AC-12, No. 6, December 1967, pp. 752-755.

40. C. A. Desoer and R. A. Baker, "On scalar products of signals passing through memoryless nonlinearities with delay," ERL Int. Tech. Memorandum M198, December 22, 1966).

41. C. A. Desoer and R. A. Baker, "Asymtotic stability in the large of a class of single-loop feedback systems," SIAM Jour. of Control, Vol. 6, No. 1, 1968, pp. 1-8, (also ERL Int. Tech. Memorandum M201, January 31, 1967).

42. P. Falb and E. Polak, "Conditions for optimality," Chapter 13 in System Theory, L. A. Zadeh and E. Polak Eds. Mc Graw-Hill, 1968.

43. L. Forys and P. P. Varaiya, "Perturbations of optimal and sub-optimal control problems," ERL Int. Tech. Memorandum M206, March 8, 1967.

44. J. P. Jacob and E. Polak, "On finite dimensional approximations to a class of games," Jour. Math. Anal. and Appl., Vol. 21, No. 2, February 1968, pp. 287-303, (also ERL Int. Tech. Memorandum M211, April 6, 1967).

45. L. J. Forys, "On the continuity of closed loop feedback relations," IEEE Transactions on Automatic Control, Vol. AC-12, No. 6, Dec. 1967.

46. E. Polak, "Linear time-invariant systems," Chapter 5 in System Theory, L. A. Zadeh and E. Polak Eds., McGraw-Hill, 1968.

47. E. Polak and E. J. Messerli, "Second order conditions of optimality for constrained optimizaion problems in finite dimensional spaces," presented at Internation Conference on System Sciences, Honolulu, January, 1968. (also ERL Int. Tech. Memorandum M224, September 22, 1967).

48. C. A. Desoer and M. Y. Wu, "Stability of linear time-invariant systems," IEEE Trans. Circuit Theory, Vol. CT-15, No. 3, pp. 245-250, Sept. 1968.

49. E. Polak, "Necessary conditions of optimality in control and programing," Proc. AMS Summer Seminar on the Math. of the Decision Sciences, Stanford University, July - August 1967.

50. E. Polak, "An algorithm for computing the Jordan canonical form of a matrix," submitted to ACM Journal, (also ERL Int. Tech. Memorandum M223, October 17, 1967).

51. E. Polak and M. Deparis, "An algorithm for minimum energy control with convex constraints," in press IEEE Trans. on Automatic Control, (also ERL Int. Tech. Memorandum M226, November 1, 1967)

52. C. A. Desoer and M. Y. Wu, "Stability of multiple-loop feedback linear time-invariant systems," Jour. of Math. Anal. and Applications, Vol. 23, No. 1, July 1968, pp. 121-129. (also ERL Int. Tech. Memorandum M226, November 8, 1967).

53. C. A. Desoer and M. Y. Wu, "Stability of linear time-invariant feedback systems," Princeton Conference, December 1967.

54. E. Polak and E. J. Messerli, "On second order necessary conditions of optimality," in press in the SIAM Jour. on Control, (also ERL Int. Tech. Memorandum M234, December 28, 1967).

55. L. A. Zadeh, "Fuzzy algorithms," Information and Control, Vol 12, pp. 94-102, Feb., 1968, (also ERL Int. Tech. Memorandum M236, Jan. 16 1968).

56. P. Varaiya, "Differential Games of Fixed Duration," presented at the Joint Automatic Control Conference.

57. E. Polak, "On the removal of ill conditioning effects in the computation of optimal controls," presented at the International System Dynamics Symposium, Sidney, Australia, August 1968. (also ERL Int. Tech. Memorandum M235, January 9, 1968). Automatica (to appear).

58. L. A. Zadeh, "Pattern recognition, abstraction and fuzzy algorithms," to appear Proceedings of International Conf. On Methodologies of Pattern Recognition, January 1968, Hawaii.

59. C. A. Desoer and L. J. Forys, "A note concerning observable but not controllable modes, "Correspondence Item in IEEE Transaction on C. T., Vol. CT-16, No. 1, February 1969.

60. E. Polak, "On primal and dual methods for solving discrete optimal control problems," presented at the Second International Conference on Computing Methods in Optimization Problems.

61. C. A. Desoer, "An extension to the circle criterion," IEEE Trans. on Automatic Control, AC-13, No. 5, Oct. 1968, pp. 587-588.

62. L. J. Forys and P. P. Varaiya, "The ϵ -capacity of classes of unknown channels," Information and Control, Vol. 14, No. 4, April, 1969.

63. M. Y. Wu, "Comments on, "Stability of feedback systems containing a single odd monotonic nonlinearity," Correspondence Section in the IEEE Transactions on Automatic Control, Vol. AC-13, No. 6, pp. 756-757, December, 1968.

64. E. J. Messerli, "On second-order necessary conditions for constrained minimal," Ph.D. Dissertation, Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, September 1968.

65. C. A. Desoer and M. Y. Wu, "L^{ρ}-Stability ($1<p<\infty$) of nonlinear time-varying feedback systems," presented at the 6th Annual Allerton Conference, Urbana, Illinois and also accepted by SIAM Journal on Control.

66. C. A. Desoer and M. Y. Wu, "Stability of a nonlinear time-invariant feedback system under almost constant inputs," Second Asilomar Conference on Circuits and Systems, 1958; published in Automatic Vol. 5, 2, p. 231-233, March 1969. (also ERL Int. Tech. Memorandum M249.)

67. L. A. Zadeh, "The concepts of system, aggregated state in system theory," Chapter I in System Theory, McGraw-Hill Company, 1968.

68. E. J. Messerli "Second-order necessary conditions for constrained minimization problems in linear topological spaces, submitted to SIAM J. on Control.

69. C. A. Desoer, "Slowly varying system $x = A(t)x$" submitted as a Letter-to the Editor of the IEEE Trans. on Automatic Control.

70. L. A. Zadeh, "Toward a theory of fuzzy systems," Report ERL-69-2, June 1969.

71. R. M. Bass, "Testing function techniques in system identification," Report ERL-69-3, 12 May 1969.

72. K. Y. Wong and E. Polak, "Identification of linear discrete time systems using the instrumental variable method," IEEE Trans. on Automatic Control, Vol. AC 12, No. 6, 1967, pp. 707-718.

73. P. Falb and E. Polak, "Conditions for optimality," Chapter 13, "System Theory," L. A. Zadeh, E. Polak, Eds., McGraw-Hill, 1968.

74. J. P. Jacob and E. Polak, "On finite dimensional approximations to a class of games", J. Math. Anal. & Appl. Vol. 21, No. 2 Feb. 1968 pp. 287-303.

75. J. P. Jacob and E. Polak, "On a class of pursuit-evasion problems," IEEE Trans. Vol. AC 12, No. 4, 1967.

76. E. Polak, "Computational methods in discrete optimal control and nonlinear programming a unified approach," University of California ERL -M261 Memorandum, February 1969, To be published under the title "Notes On Computational Methods In Optimization", Van Nostrand, N. Y., 1969.

77. E. Polak, "On the convergence of optimization algorithms," Revue Francaise d'Informatique et de Recherche Operationelle, Serie Rouge No. 16, pp. 17-34, 1969.

78. E. Polak and G. Ribiere, "Note sur la convergence de methodes de directions conjuguees,"R.I.R.O., No. 16-R, 1969.

79. C. A. Desoer and M. Y. Wu, "Input-output properties of multiple-input multiple-output discrete systems: Part I," submitted to Jour. of the Franklin Inst. (also ERL Int. Tech. Memorandum M259, May, 1969) Part II, Jour. Franklin Inst. (also ERL Int. Tech. Memorandum M262, June, 1969.

80. E. Messerli, "A hybrid conjugate gradient steepest descent algorithms for unconstrained minimization," submitted to Jour. of Optimization, Theory & Applications.

81. C. A. Desoer and M. K. Inan, "Optimization of nonlinear characteristics," presented at the 7th Annual Allerton Conf. on Computer & System Theory, October 8-10, 1969, Allerton, Illinois, pp. 166-168. Accepted for publication by Journal of Mathematical Analysis and Applications.

82. G. Meyer and E. Polak, "A decomposition algorithm for solving a class of optimal control problems," submitted to Jour. of Math. Anal. and Applications.

83. C. A. Desoer and M. Y. Wu, "Input-output properties of linear discrete systems," presented at the 7th Annual Allerton Conf. on Computer & System Theory, October 8-10, 1969, Allerton Illinois, pp. 605-609.

84. G. Meyer and E. Polak, "Abstract models for the synthesis of optimization algorithms," submitted to SIAM Jour. on Control.

85. M. Y. Wu and C. A. Desoer, "Input output properties of nonlinear discrete systems," Proc. 7th Allerton Conference (October 1969) Allerton, Illinois, pp. 610-615.